Seminar of the Chair of Optimization and Control
under prof. Stanisław Migórski
summer semester 2022-2023, Thursday, 10:15-11:45

March 9, 2023
Anna Kulig, A Fixed Point Argument for Variational-Hemivariational Inequalities

ABSTRACT: In this paper we provide a new approach in the study of a variational-hemivariational inequality in Hilbert space, based on the theory of maximal monotone operators and the Banach fixed point theorem. First, we introduce the inequality problem we are interested in, list the assumptions on the data and show that it is governed by a multivalued maximal monotone operator. Then, we prove that solving the variational-hemivariational inequality is equivalent to finding a fixed point for the resolvent of this operator. Based on this equivalence result, we use the Banach contraction principle to prove the unique solvability of the problem. Moreover, we construct the corresponding Picard, Krasnoselski and Mann iterations and deduce their convergence to the unique solution of the variational-hemivariational inequality.

The talk is based on the paper: Rong Hu, Mircea Sofonea and Yi-Bin Xiao, A Fixed Point Argument for Variational-Hemivariational Inequalities, Carpatian J. Math. 38 (2022) 573-581.

March 16, 23, 30, 2023
Krzysztof Bartosz, Noncoercive hyperbolic variational-hemivariational inequalities with an application to contact problem

ABSTRACT: In this paper we study the solvability of a coupled system which consists of a hyperbolic variational-hemivariational inequality and an equation. The inequality is in the form of a second order evolution inclusion involving both convex and Clarke's subdifferentials. Unlike most of inequality problems in literature, the one here is noncoercive due to the absence of a leading operator acting on the first order derivative of solutions with respect to time. As a result, the known surjectivity theorem for evolution inclusion of bounded and coercive L-pseudomonotone operator is ineffective to our problem. Based on the Rothe method and monotone operator theory, we establish an existence theorem for the coupled system. Moreover, an application to contact problem of beam is given.

The talk is based on the paper: Zijia Peng, Sheng Huang, Cuiming Ma, Noncoercive hyperbolic variational-hemivariational inequalities with an application to contact problem, submitted to Nonlinear Analysis: Real World Applications.

April 13, 20, 2023
Jan Słaboń, Primal and Dual Formulation of Plastic Flow Models

ABSTRACT: Elasto-plasitic matrerial is one that undergo permanent deformations when certain value of stress have been exeeded. During the seminar we will introduce basic assumptions done in order to transcribe this behawior into mathematic framework. We will go through two equivalent flow laws to at the end introduce two descriptions, in the form of variational inequalities, based on those laws.

The talk is based on Weimin Han, Daya Reddy, Computational Plasticity: The Variational Basis and Numerical Analysis, 1995

May 11, 2023
Anna Valette, Operator śladu na rozmaitościach subanalitycznych

ABSTRACT: Pokażemy, że jeśli $M \subset \mathbb{R}^n$ jest ograniczoną subanalityczną rozmaitością spełniajacą warunek: dla każdego $x_0 \in \overline{M}$ istneje $\varepsilon > 0$ takie, że $B(x_0, \varepsilon) \cap M$ jest spójne, to przestrzeń $\mathcal{C}^\infty(\overline{M})$ jest gęsta w przestrzeni Sobolewa $W^{1,p}(M)$ dla dostatecznie dużych p. Obserwacja ta jest kluczowa w konstrukcji operatora śladu.

May 18, 2023
Michał Jureczka, Application of machine learning to physical simulations and contact mechanics

ABSTRACT: In recent years, many approaches have been proposed to integrate machine learning techniques into physical simulations. Physics-informed neural networks (PINNs) are one such method that can leverage the knowledge of any physical laws along with measurement data in the learning process. Deep learning models called autoencoders can be used to perform sophisticated model reduction to increase simulation speed. Graph neural networks (GNNs) are another family of models that can operate on graph-structured physical data that can represent molecules and their interactions, or solid bodies as meshes. In this talk, we will present a survey of some applications of machine learning to physical simulations and mechanical contact problems.

May 25, 2023
Justyna Porzycka, Some applications of the topological space β_D

ABSTRACT: The aim of this talk is to present the concept of β_D space, it's algebraic structure and some applications in the branch of combinatorics known as Ramsey Theory.

June 1, 2023
Piotr Bartman, Numerical analysis of a family of simultaneous distributed-boundary mixed elliptic optimal control problems and their asymptotic behavior through a commutative diagram and error estimates

ABSTRACT: This seminar is based on the paper with the same title. We consider a family of simultaneous distributed-boundary optimal control problems (P_α) on the internal energy and the heat flux for a system governed by a mixed elliptic variational equality with a parameter $\alpha > 0$ (the heat transfer coefficient on a portion of the boundary of the domain) and a simultaneous distributed-boundary optimal control problem (P) governed also by an elliptic variational equality with a Dirichlet boundary condition on the same portion of the boundary. We formulate discrete approximations $(P_{h\alpha})$ and (P_h) of the optimal control problems (P_α) and (P) respectively, for each $h > 0$ and for each $\alpha > 0$, through the finite element method with Lagrange's triangles of type 1 with parameter h (the longest side of the triangles). The goal is to study the convergence of this family of discrete simultaneous distributed-boundary mixed elliptic optimal control problems $(P_{h\alpha})$ when the parameters α goes to infinity and the parameter h goes to zero simultaneously.

June 15, 2023
Domingo Tarzia, Optimal Control Problems for Elliptic Hemivariational Inequalities

ABSTRACT: We consider a bounded domain Ω in $\mathbb{R}^d$ whose regular boundary Γ consist of the union of three disjoint portions Γ_i, $i = 1, 2, 3$ with $meas(\Gamma_i) > 0$. We formulate the following nonlinear elliptic problem with mixed boundary conditions [3]:

$$-\Delta u = g \ \text{ in } \ \Omega, \quad u\big|_{\Gamma_1} = 0, \quad -\frac{\partial u}{\partial n}\big|_{\Gamma_2} = q, \quad -\frac{\partial u}{\partial n}\big|_{\Gamma_3} \in \alpha\, \partial j(u), \qquad (1)$$

where α is a positive constant, $g \in L^2(\Omega)$, $q \in L^2(\Gamma_2)$ and the function $j\colon \Gamma_3 \times \mathbb{R} \to \mathbb{R}$, called a superpotential (nonconvex potential), is such that $j(x, \cdot)$ is localy Lipschitz for a.e. $x \in \Gamma_3$ and not necessary differentiable. Such multivalued condition on Γ_3 is denoted for a nonmonotone relation expressed by the generalized gradient of Clarke [2]. The weak formulation of (1) is given by the elliptic hemivariational inequality [3, 5]:

$$\text{find } \ u \in V_0 \ \text{ such that } \ a(u, v) + \alpha \int_{\Gamma_3} j^0(u; v)\, d\Gamma \geq L(v), \ \ \forall v \in V_0, \qquad (2)$$

where j^0 represent the generalized (Clarke) directional derivative, $a(u, v) = \int_\Omega \nabla u\, \nabla v\, dx$, $L(v) = \int_\Omega gv\, dx - \int_{\Gamma_2} qv\, d\gamma$ and $V_0 = \{v \in H^1(\Omega) : v = 0 \ \text{ on } \ \Gamma_1\}$.

We formulate for each $\alpha > 0$, different optimal control problems (C_α), on the internal energy g and the heat flux q, for quadratic cost functional and we prove existence results for the optimal solutions (see [1, 4]). We also consider a problem as (1), with a Dirichlet condition on Γ_3 and we formulate similar optimal control problems (C), on control variables g and q. We obtain, convergence results for optimal controls and system states (C_α) to the corresponding optimal control and system state (C), when the parameter α goes to infinity.

Literatura

[1] C. M. Bollo, C. M. Gariboldi, D. A. Tarzia, *Simultaneous distributed and Neumann boundary optimal control problems for elliptic hemivariational inequalities.* J. Nonlinear Variational Analysis, 6 (5) , 535-549, 2022.

[2] F. H. Clarke, *Optimization and Nonsmooth Analysis*, Wiley, New York, 1983.

[3] C. M. Gariboldi, S. Migorski, A. Ochal, D. A. Tarzia, *Existence, comparison, and convergence results for a class of elliptic hemivariational inequalities.* Appl. Math. Optim., 84 (Suppl 2), S1453-S1475, 2021.

[4] C. M. Gariboldi, D. A. Tarzia, *Distributed optimal control problems for a class of elliptic hemivariational inequalities with a parameter and its asymptotic behavior.* Communications in Nonlinear Science and Numerical Simulation, 104 No.106027, 1-9, 2022.

[5] S. Migorski, A. Ochal, M. Sofonea, *Nonlinear Inclusions and Hemivariational Inequalities*, Springer, New York, 2013.