

**Seminar of the Chair of Optimization and Control**  
**under prof. Stanisław Migórski**  
**summer semester 2021-2022, Thursday, 10:15-11:45**

**March 3, 2022**

Wiktor Prządka, Variational approach to impulsive differential equations

ABSTRACT: We consider a linear Dirichlet problem and the solutions are found as critical points of a functional. We also study the nonlinear Dirichlet impulsive problem.

The talk is based on the paper: Variational approach to impulsive differential equations, Juan J. Nieto, Donal O'Regan, 25 October 2007, Nonlinear Analysis: Real World Applications 10 (2009) 680–690

**March 10, 2022**

Michał Jureczka, Wprowadzenie do komputerowych symulacji zjawisk mechaniki kontaktowej

**March 17, 2022**

Krzysztof Bartosz, A differential variational inequality in the study of contact problems with wear

ABSTRACT: We start with a mathematical model which describes the sliding contact of a viscoelastic body with a moving foundation. The contact is frictional and the wear of the contact surfaces is taken into account. We prove that this model leads to a differential variational inequality in which the unknowns are the displacement field and the wear function. Then, inspired by this model, we consider a general differential variational inequality in reflexive Banach spaces, governed by four parameters. We prove the unique solvability of the inequality as well as the continuous dependence of its solution with respect to the parameters. The proofs are based on arguments of monotonicity, compactness, convex analysis and lower semicontinuity. Then, we apply these abstract results to the mathematical model of contact for which we deduce the existence of a unique solution as well as the existence of optimal control for an associated optimal control problem. We also present the corresponding mechanical interpretations.

The talk will be based on the paper: Tao Chen, Nan-Jing Huang and Mircea Sofonea, A differential variational inequality in the study of contact problems with wear, to be published in Nonlinear Analysis: Real World Applications.

**March 24, 31, April 7, 2022**

Paweł Goliszewski, Zastosowanie metody elementów wirtualnych do eliptycznych

nierówności hemiwariacyjnych

ABSTRACT: W ramach prezentacji zostanie omówiona pokrótce metoda elementów wirtualnych, będąca uogólnieniem metody elementów skończonych, oraz jej zastosowanie do wybranych problemów mechaniki kontaktowej opisywanych przez eliptyczne nierówności hemiwariacyjne. Zostaną przedstawione sformułowania problemów oraz analiza błędu metody.

Referat opiera się o publikację "The virtual element method for general elliptic hemivariational inequalities" autorstwa Fei Wang, Bangmin Wu, Weimin Han, opublikowaną w "Journal of Computational and Applied Mathematics", dostępną pod adresem: <https://doi.org/10.1016/j.cam.2020.113330>

**April 21, 28, 2022**

Paweł Szafranec, Attractors for reaction-diffusion inclusions in unbounded domain

ABSTRACT: I will show existence of  $L^2$  global attractor for reaction-diffusion inclusion on  $R^N$  with non-linear multivalued right hand side, then talk about possible ways to obtain higher regularity of attractor in  $L^p$ ,  $H^1$  spaces and some other properties.

Basic refence for a single-valued problem is: B. Wang, Attractors for reaction-diffusion equations in unbounded domains, Physica D 128 (1999), 41-52.

**May 5, 2022**

Michał Jureczka, Wykorzystanie grafowych sieci neuronowych w symulacjach zjawisk mechaniki kontaktowej

ABSTRACT: W referacie przedstawię bieżące wyniki badań mających na celu wykorzystanie nowych metod sztucznej inteligencji jako alternatywy dla Metody Elementów Skończonych.

**May 12, 19, 2022**

Anna Kulig, Evolutionary problems driven by variational inequalities

ABSTRACT: In this paper we introduce the differential system obtained by mixing an evolution equation and a variational inequality ((EEVI), for short). First, by using KKM theorem and monotonicity arguments, we prove the superpositional measurability and upper semicontinuity for the solution set of a general variational inequality. Then we establish that the solution set of (EEVI) is nonempty and compact. Our approach is based on the theory of semigroups, Filippov implicit function lemma and fixed point theory for set-valued mappings.

The talk will be based on the paper: Z. Liu, S. Zeng and D. Motreanu, Evolutionary problems driven by variational inequalities, Journal of Differential Equations,

Volume 260, Issue 9, 5 May 2016, Pages 6787-6799,  
<https://doi.org/10.1016/j.jde.2016.01.012>

**May 26, 2022**

Krzysztof Winowski,  $H^1$  versus  $C^1$  local minimizer

ABSTRACT: The aim of the talk is to present the result by Nirenberg and Brezis obtained in 1993 concerning the relation between  $H^1$  and  $C^1$  local minimizers. This was the first result of this type. I will talk about the technique used in the proof and about application of this theorem in PDEs.

The talk is based on the paper: Haim Brezis, Louis Nirenberg,  $H^1$  versus  $C^1$  local minimizers, C. R. Acad. Sci. Paris, t. 317, Serie I, p. 465-472, 1993

**June 2, 9, 2022**

Piotr Bartman, Numerical approximation of an electro-elastic frictional contact problem modeled by hemivariational inequality

ABSTRACT: We will based on the paper with the same title (Wei Xu et al. 2020). In this seminar, an electro-elastic frictional contact problem is studied numerically as a hemivariational inequality. Convergence of the Galerkin approximation for the hemivariational inequality is proved, and C  a's type inequalities are derived for error estimation. The results are applied to the electro-elastic contact problem, and an optimal order error estimate is deduced for linear element approximation. Finally, numerical examples are reported.