# Seminar of the Chair of Optimization and Control under prof. Stanisław Migórski summer semester 2020-2021, Thursday, 10:15-11:45

#### February 25, 2021

Anna Valette, Trace operators on bounded subanalytic manifolds

ABSTRACT: Firstly, we will show that if a subanalytic bounded submanifold  $M \subset \mathbb{R}^n$  is normal i.e., if M is connected at every point of its frontier, then, for every sufficiently large p (not infinite), the set  $\mathcal{C}^{\infty}(\overline{M})$  is dense in the Sobolev space  $W^{1,p}(M)$ . The condition of being normal is proved to be necessary. This result, which generalizes the famous theorem which is known on manifolds with Lipschitz boundary, is useful to define the trace, and then we will prove that, if A is a subanalytic subset of  $\overline{M} \setminus M$  then the operator  $\mathcal{C}^{\infty}(\overline{M}) \ni u \mapsto u_{|A|}$  is bounded, if A is endowed with the Hausdorff measure (for p large enough).

#### March 4, 2021

Anna Ochal, A class of impulsive evolution inclusions with a history-dependent operator

ABSTRACT: We consider nonlinear first order impulsive evolution inclusions with a history-dependent operator. We provide results on the existence of a solution to the Cauchy problem and the compactness of the solution set. A talk is based on the paper with S. Migórski.

#### March 11, 2021

Krzysztof Bartosz, Equivalent projectors for virtual element methods

ABSTRACT: In the original virtual element space with degree of accuracy k, projector operators in the  $H^1$ -seminorm onto polynomials of degree  $\leq k$  can be easily computed. On the other hand, projections in the  $L^2$  norm are available only on polynomials of degree  $\leq k-2$  (directly from the degrees of freedom). Here, we present a variant of the virtual element method that allows the exact computations of the  $L^2$ projections on all polynomials of degree  $\leq k$ . The interest of this construction is illustrated with some simple examples, including the construction of three-dimensional virtual elements, the treatment of lower-order terms, the treatment of the right-hand side, and the  $L^2$  error estimates.

The talk will be based on the paper: B. Ahmada, A. Alsaedi, F. Brezzi, L.D. Marini, A. Russo, Equivalent projectors for virtual element methods, Computers & Mathematics with Applications, Vol. 66, 3, 2013, 376-391, https://doi.org/10.1016/j.camwa.2013.05.015.

### March 18, 2021

Krzysztof Winowski, Moser iteration technique

ABSTRACT: I will tell about the Moser iteration technique and its applications in proofs of boundedness of solutions of elliptic equations. I will also consider some current works on this topic and open problems.

# April 8, 2021

Anna Kulig, Penalty method for a class of differential hemivariational inequalities with application

The talk was based on the paper: Penalty method for a class of differential hemivariational inequalities with application, Z. Faiz, O. Baiz , H. Benaissa z and D. El Moutawakil.

## April 15, 2021

Paweł Goliszewski, Maximum Principle Based Algorithms for Deep Learning

ABSTRACT: The continuous dynamical system approach to deep learning is explored to develop alternative frameworks for training algorithms. Training is reformulated as a control problem and this allows us to formulate necessary optimality conditions in continuous time using the Pontryagin's Maximum principle (PMP). A modification of the method of successive approximations is then used to solve the PMP, giving us an alternative training algorithm for deep learning. The approach gives us new ways to solve problems associated with machine learning, such as trapping in slow manifolds and inapplicability of gradient-based methods for discrete trainable variables.

# April 22, 2021

Mateusz Curzydło, Relaxation Methods and Applications

ABSTRACT: This is a seminar about chapter 5 of book Numerical methods for nonlinear variational" by R. Głowinski. During lecture we will discuss topics as Convex Analisys and characteristics of the convex minimalization problems, Convex Functionals, Block Relaxation methods, Quadratic functionals and Relaxation Methods for Nonlinear Equations.

### April 29, 2021

Michał Jureczka, Adaptive Learning on the Grids for Elliptic Hemivariational Inequalities

ABSTRACT: This paper introduces a deep learning method for solving an elliptic hemivariational inequality (HVI). In this method, an expectation minimization problem is first formulated based on the variational principle of underlying HVI, which is solved by stochastic optimization algorithms using three different training strategies for updating network parameters. The method is applied to solve two practical problems in contact mechanics, one of which is a frictional bilateral contact problem and the other of which is a frictionless normal compliance contact problem. Numerical results show that the deep learning method is efficient in solving HVIs and the adaptive mesh-free multigrid algorithm can provide the most accurate solution among the three learning methods.

## May 6, 2021

Piotr Bartman, Numerical Analysis of a Parabolic Hemivariational Inequality for Semipermeable Media

ABSTRACT: During the seminar we will consider the numerical solution of a model problem in the form of a parabolic hemivariational inequality that arises in applications of semipermeable media. We'll start from indroducing the problem and discuss some sample numerical examples. In the paper the numerical solution of the model problem is presented and an optimal order error estimate with the use of linear finite elements is derived. To do this, the model problem is studied as a particular case of an abstract parabolic hemivariational inequality and a general fully discrete numerical method is introduced. Finally, numerical convergence results will be presented to show that they match the theoretically predicted optimal first order convergence of the linear element solutions with respect to the finite element mesh-size and the time step-size.

The talk will be based on the paper: Weimin Han, Cheng Wang, Numerical analysis of a parabolic hemivariational inequality for semipermeable media, J. Comp. App. Math., Vol 389, 2021, https://doi.org/10.1016/j.cam.2020.113326

### May 13, 2021

Jakub Ferliński, Wstęp do modelowania obiektowego

ABSTRACT: Podczas prezentacji przedstawię temat modelowania matematycznego wraz z dwoma metodami modelowania: modelowanie za pomocą proporcjonalności oraz modelowanie za pomocą podobieństwa geometrycznego. Prezentacja na podstawie rozdziału 2 z książki "A First Course in Mathematical Modelling", Frank R. Giordano, William P. Fox, Steven B. Horton

# May 20, 2021

Krzysztof Bartosz, The virtual element method for general elliptic hemivariational inequalities

ABSTRACT: An abstract framework of the virtual element method is established for solving general elliptic hemivariational inequalities with or without constraint, and a unified a priori error analysis is given for both cases. Then, virtual element methods of arbitrary order are applied to solve three elliptic hemivariational inequalities arising in contact mechanics, and optimal order error estimates are shown with the linear virtual element solutions. Numerical simulation results are reported in several contact problems; in particular, the numerical convergence orders of the lowest order virtual element solutions are shown to be in good agreement with the theoretical predictions. The talk will be based on the paper: Fei Wang, Bangmin Wu, Weimin Han, The virtual element method for general elliptic hemivariational inequalities, Journal of Computational and Applied Mathematics, 389 (2021) 113-330

# May 27, 2021

Krzysztof Winowski, The boundedness and Holder continuity of weak solutions to elliptic equations involving variable exponents and critical growth

The talk is based on a paper: K. Ho, Y.-H. Kim, P. Winkert, C. Zhang, The boundedness and Hölder continuity of solutions to elliptic equations involving variable exponents and critical growth.

### June 10, 2021

Anna Kulig, A Revisit of Elliptic Variational-Hemivariational Inequalities

ABSTRACT: In this paper, we provide an alternative approach to establish the solution existence and uniqueness for elliptic variational hemivariational inequalities. The new approach is based on elementary results from functional analysis, and thus removes the need of the notion of pseudomonotonicity and the dependence on surjectivity results for pseudomonotone operators. This makes the theory of elliptic variational-hemivariational inequalities more accessible to applied mathematicians and engineers. In addition, equivalent minimization principles are further explored for particular elliptic variational-hemivariational inequalities. Representative examples from contact mechanics are discussed to illustrate application of the theoretical results.